THE NONRELATIVISTIC LIMIT OF THE SECOND ORDER S-MATRIX ELEMENT

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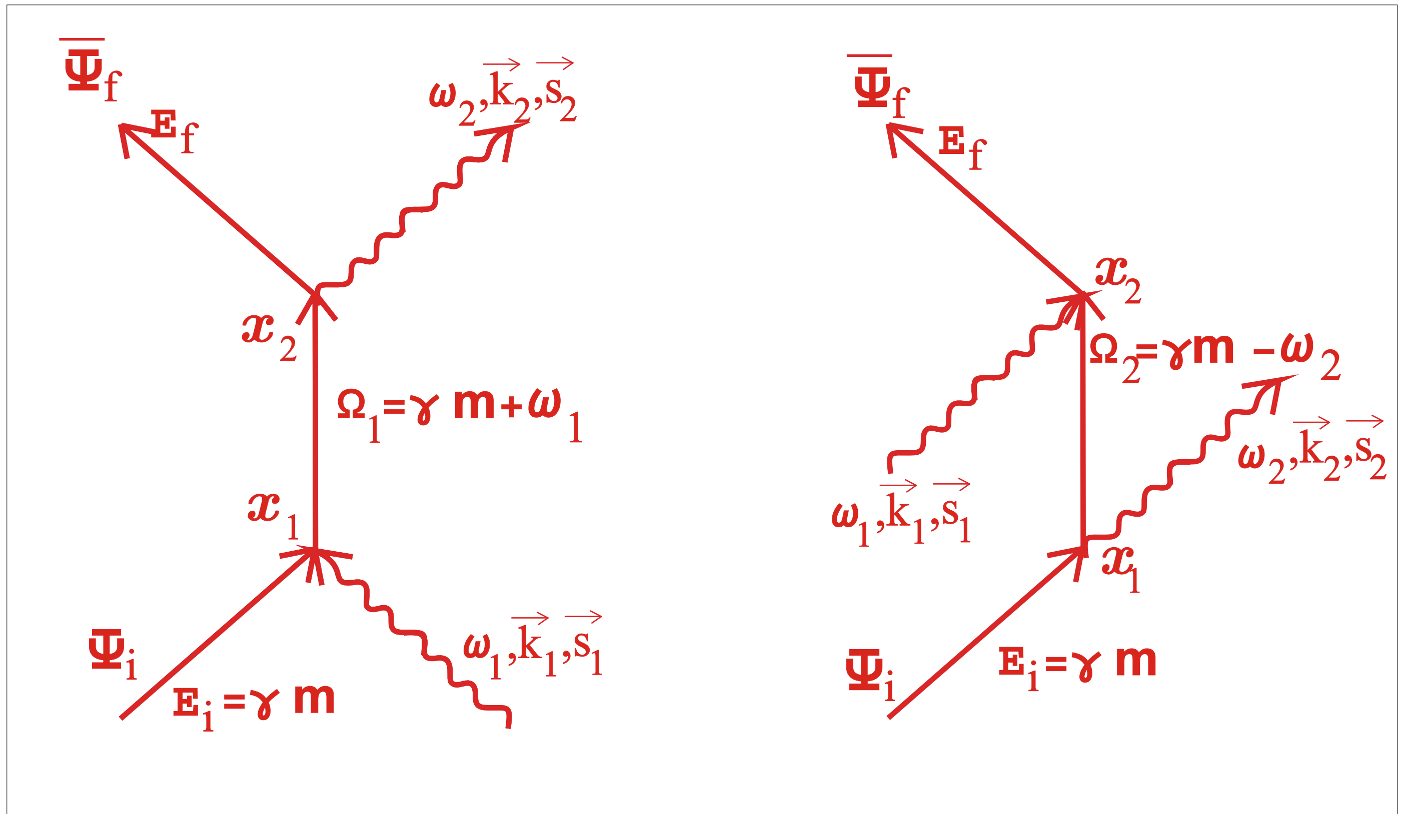
There are two Feynman diagrams for the second order amplitudes of the Compton scattering process shown in the following figure:



The matrix element is given by $M_{if} = M_{if}(\Omega_1) + M_{if}(\Omega_2)$, (1.1)

where

$\Omega_1 = \omega_1 + \gamma\, m + i\varepsilon,\quad \Omega_2 = -\omega_2 + \gamma\, m + i\varepsilon\,,\ \gamma = (1-\alpha^2 Z^2)^{\frac{1}{2}}$,

$$M_{if}(\Omega_1) = -m \lim_{\varepsilon\to 0} \mathop{S}_{n} \frac{\langle f\,|\, e^{-i\vec{k}_2\vec{r}_2}(\vec{\alpha}\,\vec{s}_2)\,|\, n\rangle\,\langle n\,|\, e^{i\vec{k}_1\vec{r}_1}(\vec{\alpha}\,\vec{s}_1)\,|\, i\rangle}{E_n - (\omega_1 + \gamma\, m + i\varepsilon)}\ , \tag{1.2}$$

$$M_{if}(\Omega_2) = -m \lim_{\varepsilon\to 0} \mathop{S}_{n} \frac{\langle f\,|\, e^{i\vec{k}_1\vec{r}_2}(\vec{\alpha}\,\vec{s}_1)\,|\, n\rangle\,\langle n\,|\, e^{-i\vec{k}_2\vec{r}_1}(\vec{\alpha}\,\vec{s}_2)\,|\, i\rangle}{E_n - (\gamma\, m - \omega_2 - i\varepsilon)}\ . \tag{1.3}$$

The sum over all intermediate states can be replaced by introducing the Green function of the Dirac equation with Coulombian field,

$$M_{fi}(\Omega_1) = -m\langle f\,|\, e^{-i\vec{k}_2\vec{r}_2}(\vec{\alpha}\,\vec{s}_2)\, G(\vec{r}_2\,\vec{r}_1;\Omega_1)\, e^{i\vec{k}_1\vec{r}_1}(\vec{\alpha}\,\vec{s}_1)\,|\, i\rangle\,, \tag{1.4}$$

$$M_{fi}(\Omega_2) = -m\langle f\,|\, e^{i\vec{k}_1\vec{r}_2}(\vec{\alpha}\,\vec{s}_1)\, G(\vec{r}_2\,\vec{r}_1;\Omega_2)\, e^{-i\vec{k}_2\vec{r}_1}(\vec{\alpha}\,\vec{s}_2)\,|\, i\rangle\,, \tag{1.5}$$

$$G(\vec{r}_2\,\vec{r}_1;\Omega) = \frac{1}{2m}\left(i\vec{\alpha}\,\nabla_2 - \beta m - \frac{\alpha Z}{r_2} - \Omega\right)\left[I + \frac{1}{2\Omega}\vec{\alpha}\left(\vec{P}_2 + \vec{P}_1\right)\right] G_0(\vec{r}_2\,\vec{r}_1;\Omega), \tag{1.6}$$

$$\left(-\frac{1}{2m}\Delta_2 - \frac{\alpha Z\Omega}{m}\frac{1}{r_2} + \frac{X^2}{2m}\right) G_0(\vec{r}_2\,\vec{r}_1;\Omega) = -\delta(\vec{r}_2 - \vec{r}_1),\ \text{with}\quad X^2 = m^2 - \Omega^2\,, \tag{1.7}$$

Taking into account that $\left(\vec{\alpha}\,\vec{s}_2\right)\left(\vec{\alpha}\,\vec{P}_2\right) = -\left(\vec{\alpha}\,\vec{P}_2\right)\vec{\alpha}\,\vec{s}_2 + 2\vec{s}_2\,\vec{P}$ and $\left(u_f^{(-)}(\vec{r}_2)\right)^+\left(\vec{\alpha}\,\vec{P} + \beta m - \frac{\alpha Z}{r_2} - E_f\right) = 0$

from eqs. (1.4-1.6) we get:

$$M_{fi} = \vec{s}_1\,\vec{s}_2\,\vartheta + s_{1j}s_{2k}\,[\Pi_{jk}(\Omega_1) + \Pi_{jk}(\Omega_2)]. \tag{1.9}$$

where:

$$\vartheta = \int_{\mathbb{R}^3} d^3r\left(u_f^{(-)}(\vec{r}_2)\right)^+ e^{i(\vec{k}_2-\vec{k}_1)\vec{r}}\,u_i(\vec{r}) = \langle f\,|\,e^{i(\vec{k}_2-\vec{k}_1)\vec{r}}\,|\,i\rangle\ , \tag{1.10}$$

$$s_{1j}s_{2k}\,\Pi_{jk}(\Omega_1) = \iint_{\mathbb{R}^3\mathbb{R}^3} d^3r_1 d^3r_2\left(u_f^{(-)}(\vec{r}_2)\right)^+ \vec{s}_2\,\vec{P}_2\,G_0(\vec{r}_2\,\vec{r}_1;\Omega_1)\,\vec{\alpha}\,\vec{s}_1\,e^{i(\vec{k}_1\vec{r}_1-\vec{k}_2\vec{r}_2)}\,u_i(\vec{r}_1), \tag{1.11}$$

$$s_{1j}s_{2k}\,\Pi_{jk}(\Omega_2) = \iint_{\mathbb{R}^3\mathbb{R}^3} d^3r_1 d^3r_2\left(u_f^{(-)}(\vec{r}_2)\right)^+ \vec{s}_1\,\vec{P}_2\,G_0(\vec{r}_2\,\vec{r}_1;\Omega_2)\,\vec{\alpha}\,\vec{s}_2\,e^{i(\vec{k}_1\vec{r}_2-\vec{k}_2\vec{r}_1)}\,u_i(\vec{r}_1), \tag{1.12}$$

According to the Hostler result [1], in our case, to the iterated Green funtion $\left[I + \frac{1}{2\Omega}\vec{\alpha}\left(\vec{P}_2 + \vec{P}_1\right)\right]G_0(\vec{r}_2\,\vec{r}_1;\Omega)$ corresponds the (1.13)

iterated Dirac wavespinor with asymptotic behavior given by a spherical incoming wave

$$u^{(-)}_{\vec{p}\mu}(\vec{r}) = \left(I + \frac{\vec{\alpha}\,\vec{P}_2}{2E_f}\right)u^{(-)}_{\vec{p}}(\vec{r})\,u_\mu(\vec{p}) \quad \text{where} \quad u^{(-)}_{\vec{p}}(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\Gamma(1+i|\nu|)\,e^{\frac{\pi|\nu|}{2}}\,e^{i\vec{p}\,\vec{r}}\,{}_1F_1\left(-i|\nu|, 1; -i\left(\vec{p}\,\vec{r} + p\,r\right)\right) \tag{1.14}$$

and $u_\mu(\vec{p})$ is a normalized Dirac spinor describing a free electron with momentum $\vec{p}$ and polarization μ. The momentum p and the parameter ν are given by relativistic kinematics

The dominant term $u^{(-)}_{\vec{p}}(\vec{r})$ of the spinor $u^{(-)}_{\vec{p}\mu}(\vec{r})$ is the solution of the Schrödinger type equation $\left(-\frac{1}{2m}\Delta - \frac{\alpha Z}{r} - E_f\right)u^{(-)}_{\vec{p}}(\vec{r}) = 0$.

$p = \sqrt{E_f^2 - m^2}$, $\nu = \frac{\alpha Z E_f}{ip}$, and $E_f = m + \omega_{2\max} - \omega_2$,

where $\omega_{2\max} = \omega_1 - I_B = \omega_1 - (1-\gamma)m$.

The ground state Dirac spinor is

$$u^+_{\mu_f}(\vec{r}_2) = \sqrt{\frac{\lambda^3}{\pi}\frac{\gamma+1}{\Gamma(2\gamma+1)}}(2\lambda r)^{\gamma-1}e^{-\lambda r}\left(1 - \frac{i\alpha Z}{1+\gamma}\frac{\vec{\alpha}\,\vec{r}}{r}\right)\chi^+_\mu \quad \text{where} \quad \chi^+_\mu = \begin{cases}(1\,0\,0\,0) & if\ \ \mu = \tfrac{1}{2}\\ (0\,1\,0\,0) & if\ \ \mu = -\tfrac{1}{2}\end{cases} \tag{1.15}$$

In the nonrelativistic approach we put γ=1 and the spin term should be neglected, a consistent procedure for not too large momentum transfers.

The Compton matrix element for any s-state may be expressed in terms of invariant amplitudes A,B,C,D and E given by the equations [2]:

$$A=\vartheta-P(\Omega_1)-P(\Omega_2),\ E=-(T(\Omega_1)+T(\Omega_2)),\ B=-(L(\Omega_1)+L(\Omega_2)),\ C=-R(\Omega_1)+S(\Omega_2),\ D=-S(\Omega_1)+R(\Omega_2), \qquad (1.16)$$

Taking into account the expression of the sixfold integral [3] involved in eqs (1.11) and (1.12) and the integral representation of the Coulomb Green's function in the momentum space, we obtained the invariant amplitudes for Compton scattering in terms of five amplitudes involving four distinct Lauricella functions of D type. Thus, the triply differential Compton scattering cross-section could be analytically expressed. Some of the amplitudes are the following [4]:

$$P(\Omega_j)=\mathcal{N}_j\frac{\lambda^{\frac{7}{2}}(\alpha ZE_f)^{\frac{3}{2}}}{d^2(\Omega_j)f(\Omega_j)g(\Omega_j)}\frac{F(2-\tau_j;1-\nu,1+\nu\ ;s(\Omega_j),p(\Omega_j);s'(\Omega_j),p'(\Omega_j))}{2-\tau_j}. \qquad (1.17)$$

$$L(\Omega_j)=8\mathcal{N}_jX_j^2\frac{\lambda^5\omega_1\omega_2}{d^3(\Omega_j)f(\Omega_j)g(\Omega_j)}\left\{\frac{\nu+1}{g(\Omega_j)}\frac{F(3-\tau_j;1-\nu,2+\nu;s(\Omega_j),p(\Omega_j);s'(\Omega_j),p'(\Omega_j))}{3-\tau_j}\right. \qquad (1.18)$$

$$\left.-\frac{\nu-1}{f(\Omega_j)}\frac{F(3-\tau_j;2-\nu,1+\nu;s(\Omega_j),p(\Omega_j);s'(\Omega_j),p'(\Omega_j))}{3-\tau_j}\right\}.$$

$$R(\Omega_j)=8\mathcal{N}_jX_j^2\frac{\lambda^5 p\omega_j(\nu+1)}{d^3(\Omega_j)f(\Omega_j)g^2(\Omega_j)}\frac{F(3-\tau_j;1-\nu,2+\nu;s(\Omega_j),p(\Omega_j);s'(\Omega_j),p'(\Omega_j))}{3-\tau_j}. \qquad (1.19)$$

where

$$f(\Omega_2)=2m\omega_1e^{-i\chi_2},\ d(\Omega_1)=2\,m\omega_1e^{-i\chi_0}, \qquad (1.20)$$

$$f(\Omega_1)=-4m\omega_{2\max}\left(1-\frac{\omega_2}{2\omega_{2\max}}+\sqrt{1-\frac{\omega_2}{\omega_{2\max}}}\right), \qquad (1.21)$$

$$d(\Omega_2)=2m\omega_2\left(\gamma+\frac{\alpha^2Z^2m}{\omega_2}+\alpha Z\sqrt{\frac{m}{\omega_2}}\sqrt{2\gamma+\frac{\alpha^2Z^2m}{\omega_2}}\right), \qquad (1.22)$$

$$g(\Omega_1)=-2m\omega_2+2\omega_2\sqrt{2m(\omega_{2\max}-\omega_2)}\cos\theta',\ g(\Omega_2)=2m\omega_1-2\omega_1\sqrt{2m(\omega_{2\max}-\omega_2)}\cos\theta'', (1.23)$$

In the case of unpolarized initial photons the triply and doubly differential cross-section are, respectively:

$$\frac{d^3\sigma}{d\omega_2 d\Omega_{\vec{k}_2} d\Omega_{\vec{p}}} = \frac{1}{2}\sum_{\vec{s}_1,\vec{s}_2} |M|^2 \frac{\omega_2}{\omega_1} r_0^2 \qquad \frac{d^2\sigma}{d\Omega_{k_2} d\omega_2} = \frac{1}{2}\int_{\Omega_p} \sum_{\vec{s}_1,\vec{s}_2} |M|^2 d\Omega \frac{\omega_2}{\omega_1} r_0^2 \tag{1.24}$$

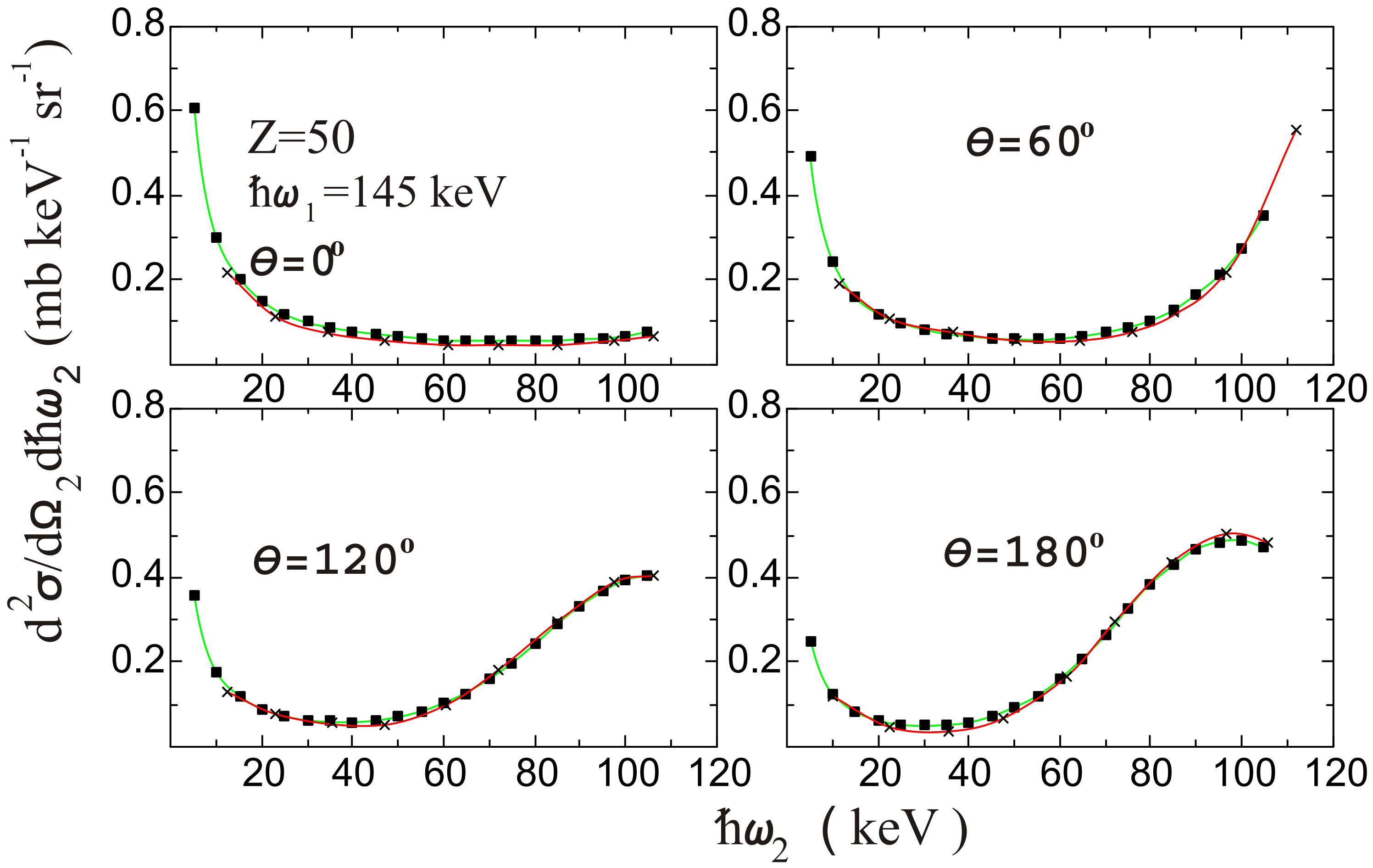
The Lauricella functions have been calculated using the integral representation:

$$F_D\left(a; b_1, b_1, b_2, b_2; a+1; x, y, x', y'\right) = \frac{1}{a}\int_0^1 \frac{\rho^{a-1}}{(1-\rho\, s(\Omega_j) + \rho^2\, p(\Omega_j))^{b_1} (1-\rho\, s'(\Omega_j) + \rho^2\, p'(\Omega_j))^{b_2}} d\rho \tag{1.25}$$

We developed a special quadrature algorithm for performing this integral, with variable step sampling [5] for optimizing the speed of the calculus, that allowed the double integration over the solid angle for obtaining the doubly differential scattering cross section and a further integration over the energies for obtaining the singly differential one.

NUMERICAL RESULTS FOR COMPTON SCATTERING ON K-SHELL ELECTRONS

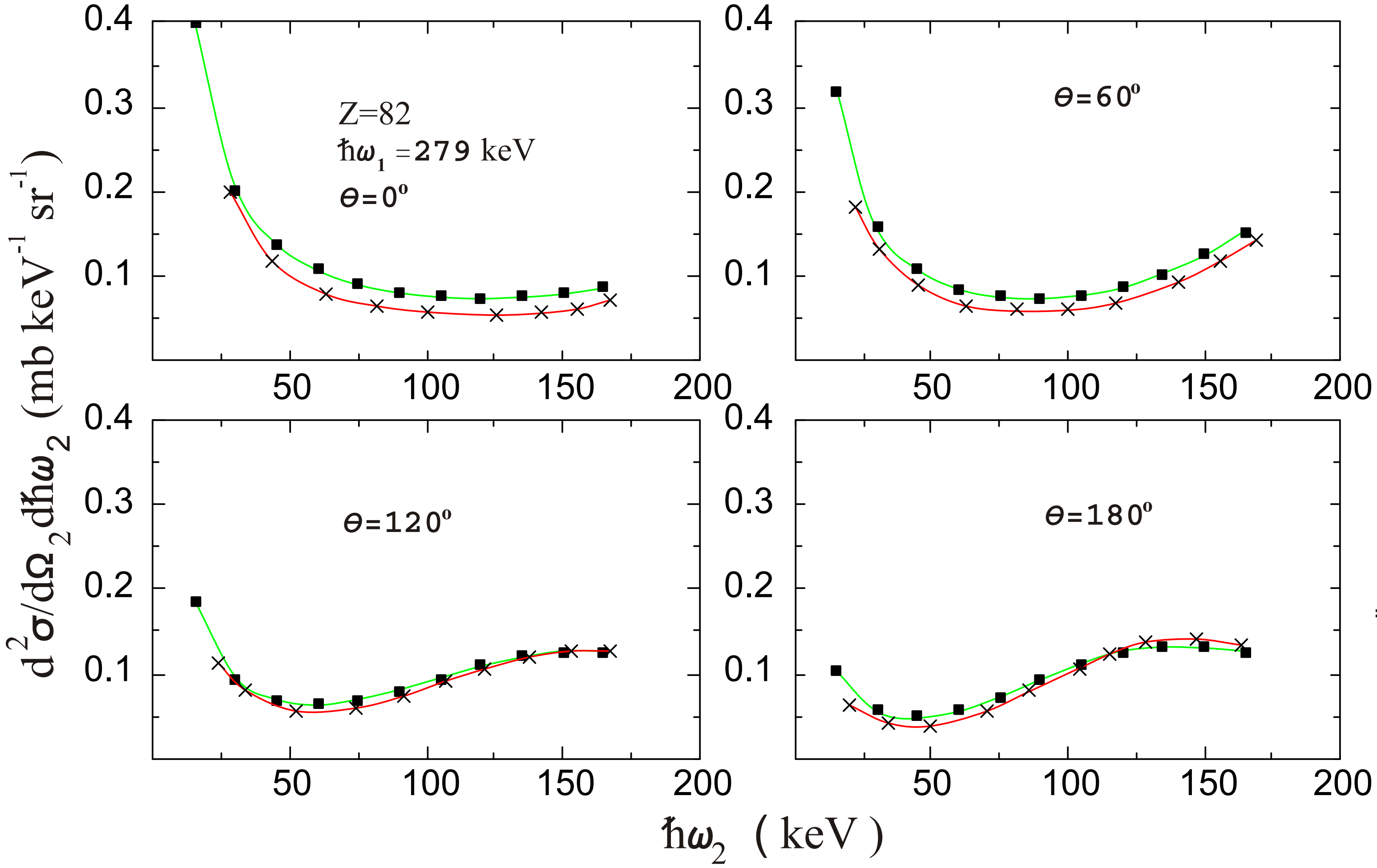
The predictions given by our formulae show a good agreement with experimental results and the full relativistic numerical evaluation of Bergstrom *et al* [6]. For intermediate and high Z targets, the agreement are within 10% for the whole spectrum and any scattering angle for photon energies below 300 keV with relativistic kinematics terms included. For z=82 and 661 keV incident photon energy the same precision was achieved for scattering angles below 60^0. We present here some of our numerical results for the doubly and singly differencial cross sections.



$\hbar\omega_2\,(keV)$	$\frac{d^2\sigma}{d\omega_2 d\Omega_2}(mb\,keV^{-1} sr^{-1})$ $\theta = 0°$	$\theta = 60°$	$\theta = 120°$	$\theta = 180°$
5	0.607434	0.491363	0.355751	0.248403
10	0.300706	0.241157	0.174822	0.122017
15	0.199208	0.158799	0.115715	0.0811422
20	0.149008	0.118439	0.0873526	0.062255
25	0.119334	0.0949583	0.071681	0.0529825
30	0.0999417	0.0800346	0.062878	0.0496354
35	0.0864524	0.0701575	0.0586961	0.0511976
40	0.0766867	0.0636398	0.0583276	0.0578709
45	0.0694444	0.0596297	0.0617188	0.0705967
50	0.064018	0.0577302	0.0693078	0.0907831
55	0.0599708	0.0578404	0.0818852	0.119958
60	0.0570288	0.0601009	0.100441	0.159208
65	0.0550226	0.0648942	0.125916	0.208401
70	0.0538594	0.072884	0.158793	0.265432
75	0.0535042	0.0850808	0.198591	0.325909
80	0.0539799	0.102924	0.243394	0.383704
85	0.0553698	0.128339	0.289723	0.43236
90	0.0578343	0.163696	0.332983	0.466803
95	0.0616421	0.21154	0.368455	0.484524
100	0.0672305	0.273911	0.392471	0.485798
105	0.0753236	0.351163	0.403246	0.473037

Doubly differential cross-sections for the scattering of 145 keV photons from a K-shell electron of tin: present work (green); Bergstrom *et al.* calculations [6] (red).

FOR TWO PHOTON PROCESSES

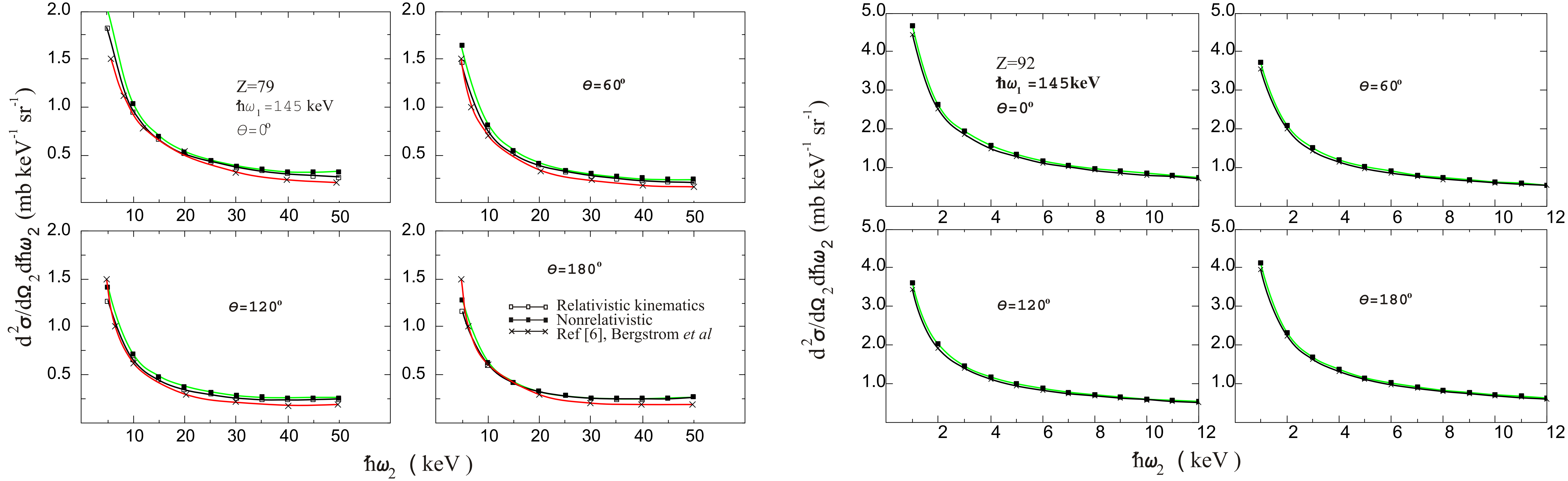


$\hbar\omega_2 (keV)$	$\frac{d^2\sigma}{d\omega_2 d\Omega_2} (mb\, keV^{-1} sr^{-1})$ $\theta = 0^\circ$	$\theta = 60^\circ$	$\theta = 120^\circ$	$\theta = 180^\circ$
15	0.400758	0.317832	0.18538	0.103364
30	0.202086	0.157802	0.0945368	0.0570044
45	0.137797	0.107551	0.0698452	0.049957
60	0.107356	0.0855669	0.0643261	0.0571207
75	0.0907079	0.075947	0.0689608	0.072949
90	0.0812492	0.0738725	0.0801595	0.0929473
105	0.0762601	0.0778256	0.0947465	0.111832
120	0.0745377	0.0875874	0.109146	0.125206
135	0.0756468	0.10341	0.120157	0.131236
150	0.0797124	0.125423	0.125952	0.130567
165	0.0875195	0.153114	0.126336	0.125075

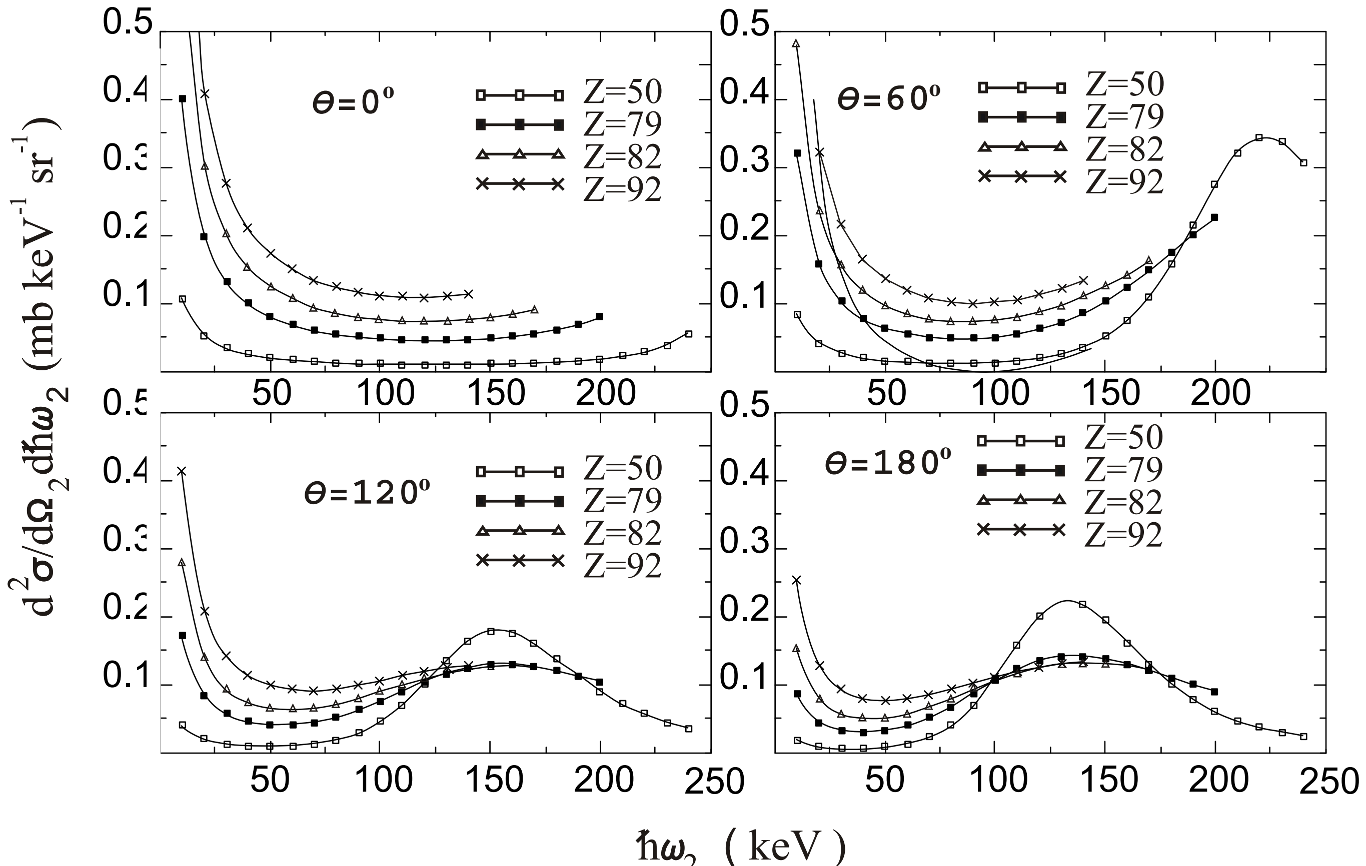
Doubly differential cross-sections for the scattering of 279 keV photons from a K-shell electron of gold: present work (green); Bergstrom *et al.* calculations [6] (red). The relativistic kinematics terms are included.

$\hbar\omega_2 (keV)$	$\frac{d^2\sigma}{d\omega_2 d\Omega_2} (mb\, keV^{-1} sr^{-1})$ $\theta = 0^\circ$		$\theta = 60^\circ$		$\theta = 120^\circ$		$\theta = 180^\circ$	
	RK	NR	RK	NR	RK	NR	RK	NR
5	1.82704	2.05744	1.45868	1.64369	1.26807	1.4217	1.16017	1.27343
10	0.952587	1.03121	0.746695	0.81639	0.651292	0.711679	0.597658	0.630006
15	0.659702	0.694907	0.509225	0.546301	0.447678	0.482441	0.413226	0.423855
20	0.513646	0.532986	0.391692	0.416894	0.349259	0.375114	0.325962	0.329492
25	0.426996	0.441618	0.322898	0.344347	0.294263	0.317513	0.279554	0.281453
30	0.370487	0.386013	0.279082	0.300532	0.262136	0.285442	0.255313	0.257816
35	0.331522	0.351172	0.250123	0.273284	0.244146	0.268415	0.245315	0.249055
40	0.30383	0.329674	0.231079	0.256408	0.236001	0.261012	0.245638	0.250148
45	0.283986	0.318114	0.21937	0.24647	0.235372	0.259952	0.254076	0.257862
50	0.270061	0.32681	0.213704	0.244895	0.240921	0.263043	0.269243	0.269272

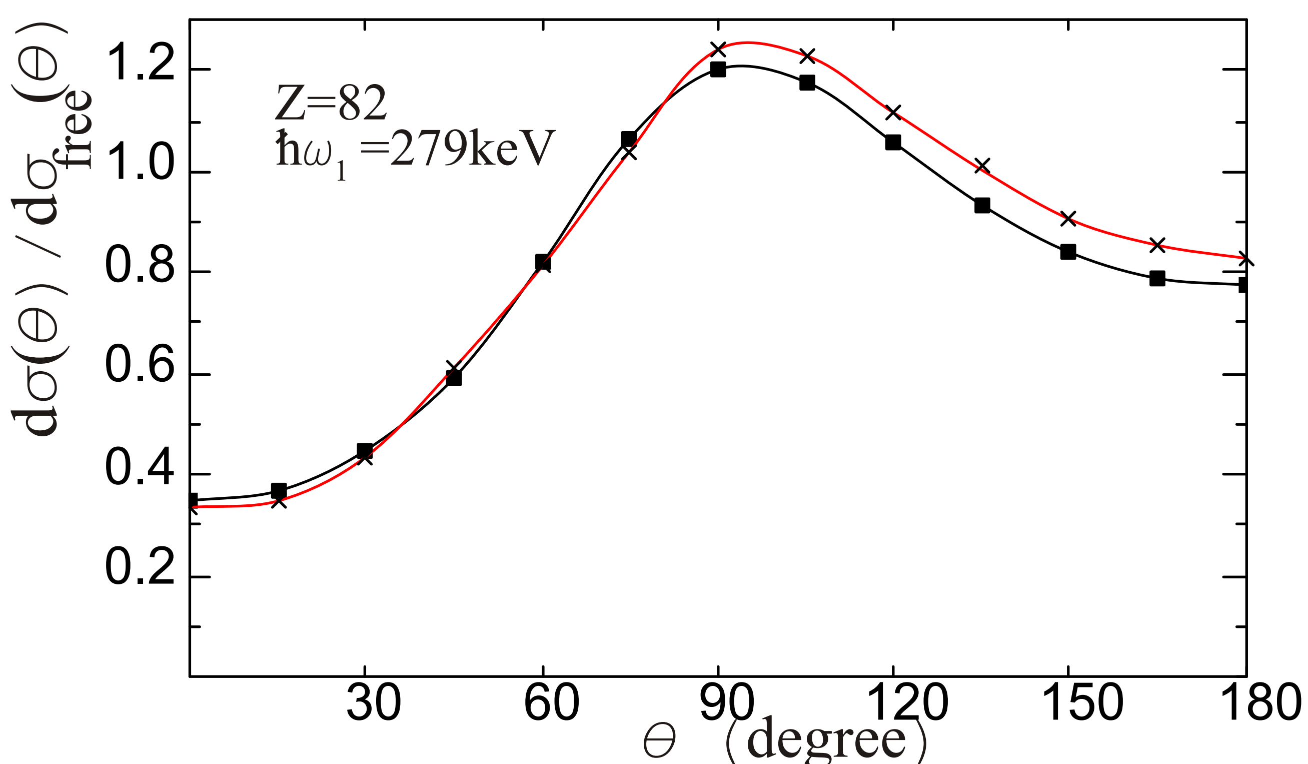
Comparison between the doubly differential cross-section with relativistic kinematics terms included (RK), and nonrelativistic(NR) for $Z=79$ and 145 keV



Doubly differential cross-sections for the scattering of 145 keV photons from a K-shell electron for gold and uranium:nonrelativistic limit (green); relativistic terms included (black) ; Bergstrom *et al.* calculations [6] (red).



Doubly differential cross-sections for the scattering of 279 keV photons from a K-shell electron of various Z elements, relativistic terms included



Singly differential cross-sections for the scattering of 279 keV photons from a K-shell electron of lead, relativistic terms included. Present work (black); full relativistic calculations of Bergstrom *et al* [6] (red)

THE NONRELATIVISTIC LIMIT OF THE RAYLEIGH SECOND ORDER S-MATRIX ELEMENT

The eqations for the matrix elements for Rayleigh scattering are similar to those that appear in the case of Compton scattering unless the final state Dirac spinor, which in the Rayleigh case corresponds to the ground state.

We point out that in the specific case of Rayleigh scattering the intermediate states belonging to continuum spectrum for which $E_n \cong E_0 + \omega$ or $E_n \cong E_0 - \omega$ have a large contribution to the matrix elements $\mathcal{M}(\Omega_1)$ and $\mathcal{M}(\Omega_2)$ respectively.

Following the method of calculation of Gavrila and Costescu [3] we get the matrix element for Rayleigh scattering written as:

$$\mathcal{M}^R_{\mu_f \mu_i} = M(\omega,\theta)(\vec{s}_1\vec{s}_2) + N(\omega,\theta)(\vec{s}_1\vec{v}_2)(\vec{s}_2\vec{v}_1) \qquad \text{, where} \qquad (2.1)$$

$$M(\omega,\theta) = [\vartheta - P(\Omega_1,\theta) - P(\Omega_2,\theta)], \qquad N(\omega,\theta) = -[Q(\Omega_1,\theta) + Q(\Omega_2,\theta)] \qquad (2.2)$$

$$P(\Omega,\theta) = 128\frac{\lambda^5 X^3}{d^4(\Omega)}\frac{F_1(2-\tau;2,2,3-\tau;x_1,x_2)}{2-\tau}, Q(\Omega,\theta) = \frac{2048\lambda^5 X^5\omega^2}{d^6(\Omega)}\frac{F_1(3-\tau;3,3,4-\tau;x_1,x_2)}{3-\tau} \qquad (2.3)$$

In the nonrelativistic limit (NR), *below the photoeffect threshold*, we get (the upper sign being for Ω_1, the lower for Ω_2):

$$d^2_{NR}(\Omega) = 4[E_0^2\omega^2 + 4\alpha^2 Z^2 m^2(\alpha^2 Z^2 m^2/2 \mp E_0\omega) + 2\alpha^2 Z^2 m^2(\alpha^2 Z^2 m^2 \mp E_0\omega)(1\mp\omega/\omega_{th})^{1/2}] \qquad (2.4)$$

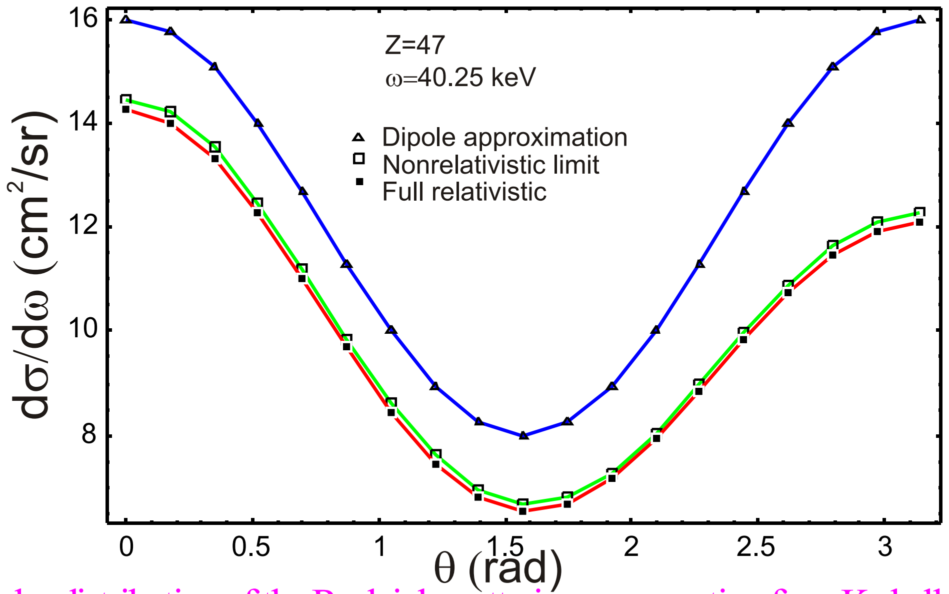
$$X_{NR}(\Omega_1) = \begin{cases} \alpha Zm(1-\omega/\omega_{th})^{1/2} & ,\omega < \omega_{th} \\ -i\alpha Zm(\omega/\omega_{th}-1)^{1/2} & ,\omega_{th} \le \omega << \omega_{pp} \end{cases} ; X_{NR}(\Omega_2) = \alpha Zm(1+\omega/\omega_{th})^{1/2}, \omega << \omega_{pp}. \qquad (2.5)$$

$$\tau_1 = \frac{\alpha Zm}{X_{NR}(\Omega_1)} = \begin{cases} (1-\omega/\omega_{th})^{-\frac{1}{2}} & ,\omega < \omega_{th} \\ i(\omega/\omega_{th}-1)^{-\frac{1}{2}} & ,\omega_{th} \le \omega << \omega_{pp} \end{cases} ; \tau_2 = \frac{\alpha Zm}{X_{NR}(\Omega_2)} = (1+\omega/\omega_{th})^{-\frac{1}{2}}, \omega << \omega_{pp} \qquad (2.6)$$

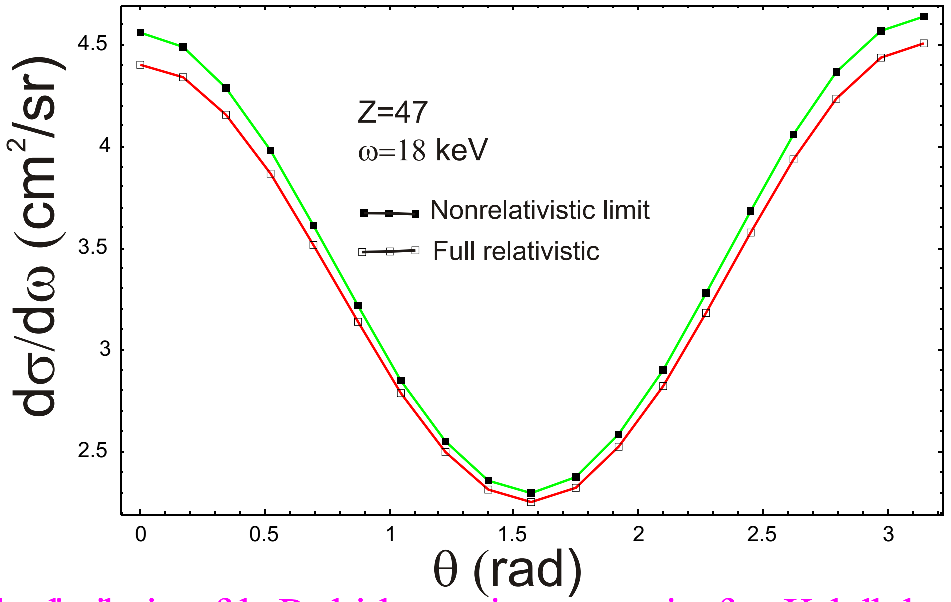
The Appell's functions variables are given by the relationships:

$$\sqrt{x_1x_2} = \xi(\Omega_1) = \frac{(\omega/\omega_{th})^2}{[(1+\sqrt{1-\omega/\omega_{th}})(1+E_0/m) - (E_0/m)(\omega/\omega_{th})]^2 + \alpha^2 Z^2\omega/\omega_{th}}, x_1 + x_2 = s = 2\left[1 - 2\frac{X^2}{m^2}\sin^2\frac{\theta}{2}\right]\xi(\Omega$$

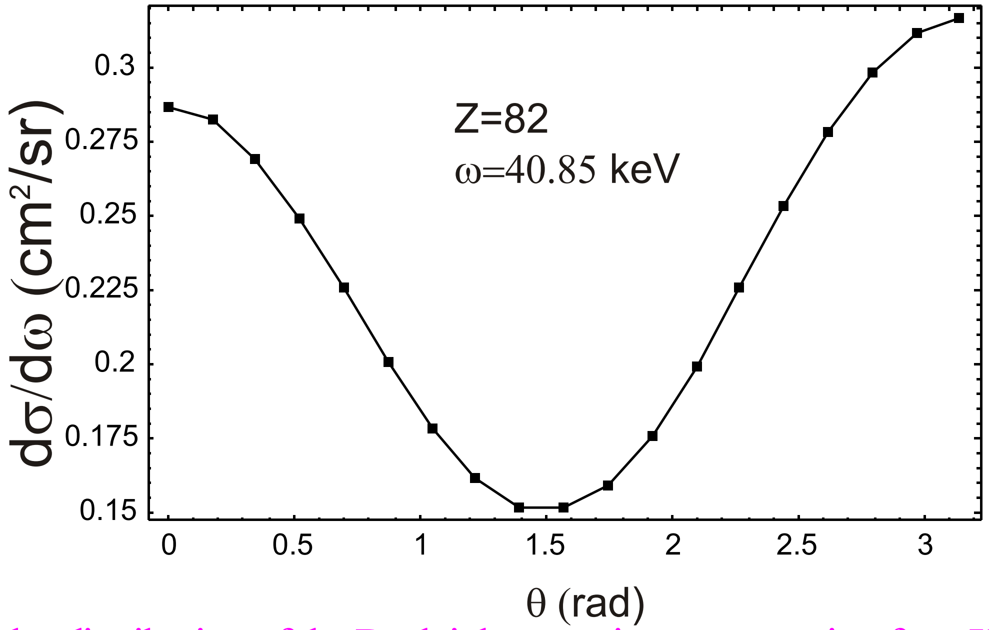
In accordance with the symmetry properties of the process we may obtain $\xi(\Omega_2)$ from (2.7) by changing the sign in the front of the energy ω i.e. $\omega \to -\omega$.



Angular distribution of the Rayleigh scattering cross section for a K-shell electron of Ag at 40.25 keV, in the nonrelativistic limit (squares), the full relativistic results (solid squares) and dipole approximation (triangles).



Angular distribution of the Rayleigh scattering cross section for a K-shell electron of Ag at 18.0 keV, in the nonrelativistic limit (solid squares) and full relativistic results (squares).



Angular distribution of the Rayleigh scattering cross section for a K-shell electron of Pb at 40.85 keV, in the nonrelativistic limit.

$$A_{\perp} = M \, , \; A_{II} = M\cos\theta - N\sin^2\theta \, , \; \frac{d\sigma}{d\omega} = \frac{r^2}{2}\left(|A_{\perp}|^2 + |A_{II}|^2\right)$$

Comparison of the Rayleigh scattering amplitude obtained by us in the nonrelativistic limit to the relativistic S-matrix results of Kissel *et. al.* [7] (RMP) for Z=47, θ=0.

ω (keV)	k_{REL}	M(present work)	M(RMP ref [7])	ε(%)
5.41	0.174544	-0.033681	-0.0335	0.54
17.43	0.562347	-0.659291	-0.649	1.58
22.10	0.713016	-3.98604	-3.713	7.35
26.0	0.838842	0.862004	0.895	-3.68

Comparison of the Rayleigh scattering amplitude obtained by us in the nonrelativistic limit to the relativistic S-matrix results of Kissel *et. al.* [7] (RMP) for Z=82, θ=0.

ω (keV)	k_{REL}	M(present work)	M(RMP ref [7])	ε(%)
5.41	0.0532576	-0.00255512	-0.00254	0.59
17.43	0.171586	-0.0275683	-0.027	2.1
40.85	0.402139	-0.190065	-0.187	1.91
59.5	0.585735	-0.657714		0.4
74.96	0.737928	-10.05	-10.536	-4.60
84.26	0.82948	1.03691	1.2155	-14.65

CONCLUSIONS

The good agreement of our calculation with the full relativistic results shows that, for the presented energies regime, the main relativistic kinematics terms are cancelled by retardation and multipoles terms, and the remaining terms have a nonrelativistic origin. Also, the spin effects are small but they settle on the right locations and number of physical resonances. For high Z values, even in the nonrelativistic limit where the nonrelativistic terms are largely dominant, all resonances and threshold energies values must be considered in accordance with their relativistic values.

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